EE 230
Lecture 35

Small Signal Models

Small Signal Analysis

Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Review from Last Time:

Solution for the example was based upon solving the nonlinear circuit for V_{OUT} and then linear zing the solution by doing a Taylor's series expansion

- *Solution of nonlinear equations very involved with two or more nonlinear devices*
- *Taylor's series linearization can get very tedious if multiple nonlinear devices are present*

Standard Approach to small-signal analysis of nonlinear networks

1. *Solve nonlinear network*
2. *Linearize solution*

Alternative Approach to small-signal analysis of nonlinear networks

1. *Linearize nonlinear devices*
2. *Replace all devices with small-signal equivalent*
3. *Solve linear small-signal network*

Review from Last Time:

Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices

2. Replace all devices with small-signal equivalent

3. Solve linear small-signal network

- **Must only develop linearized model once for any nonlinear device**

  e.g. once for a MOSFET, once for a JFET, and once for a BJT

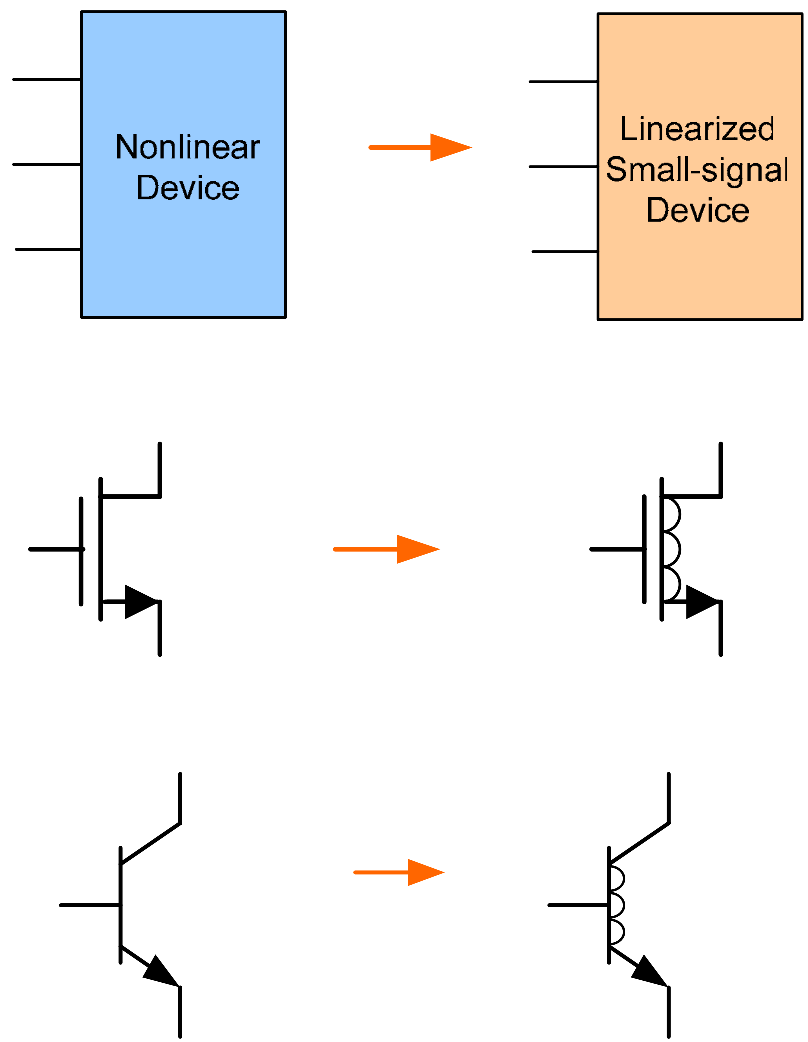
  Linearized model for nonlinear device termed “small-signal model”

  derivation of small-signal model for most nonlinear devices is less complicated than solving even one simple nonlinear circuit

- **Solution of linear network much easier than solution of nonlinear network**

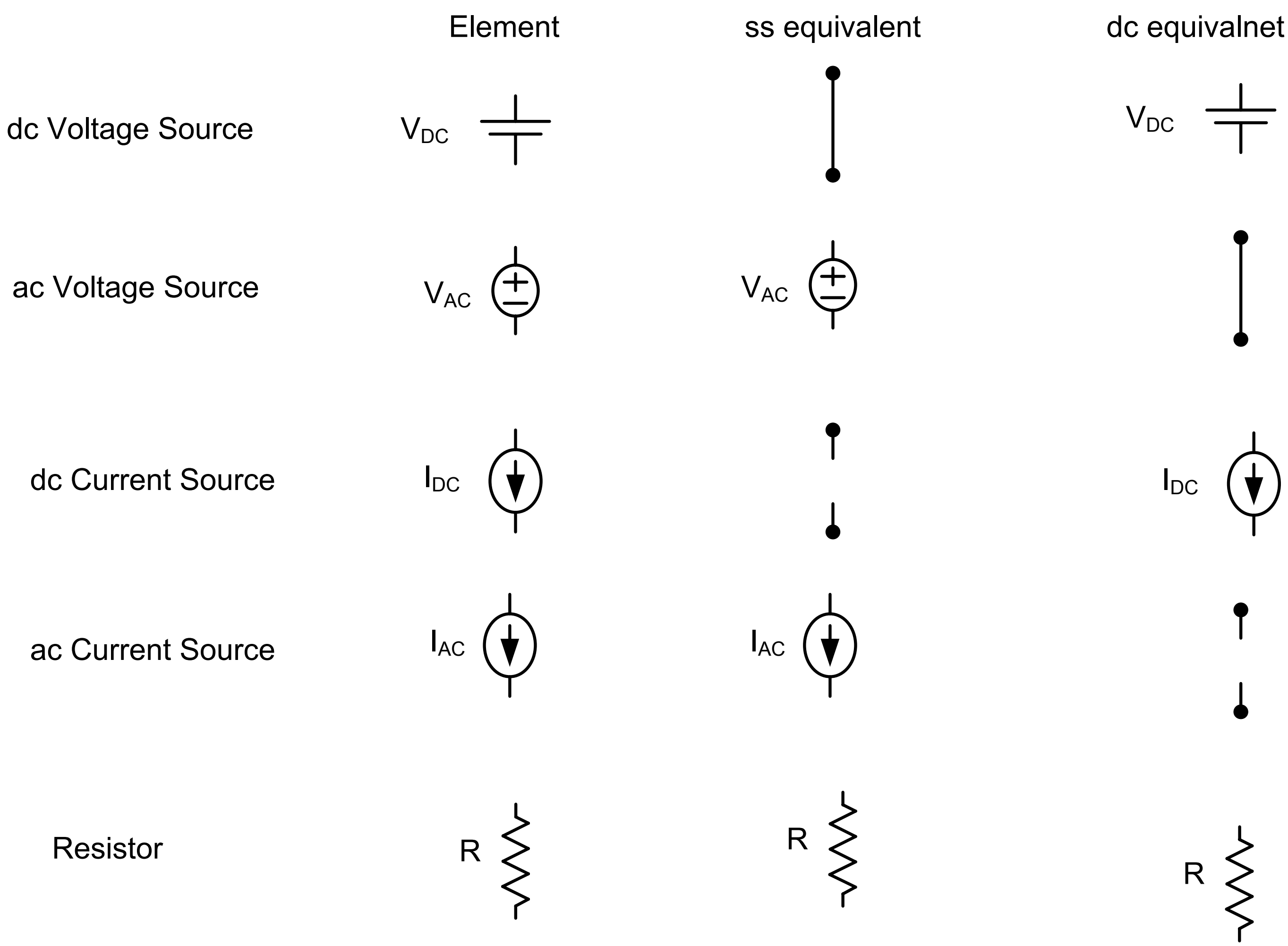
Review from Last Time:

Linearized nonlinear devices



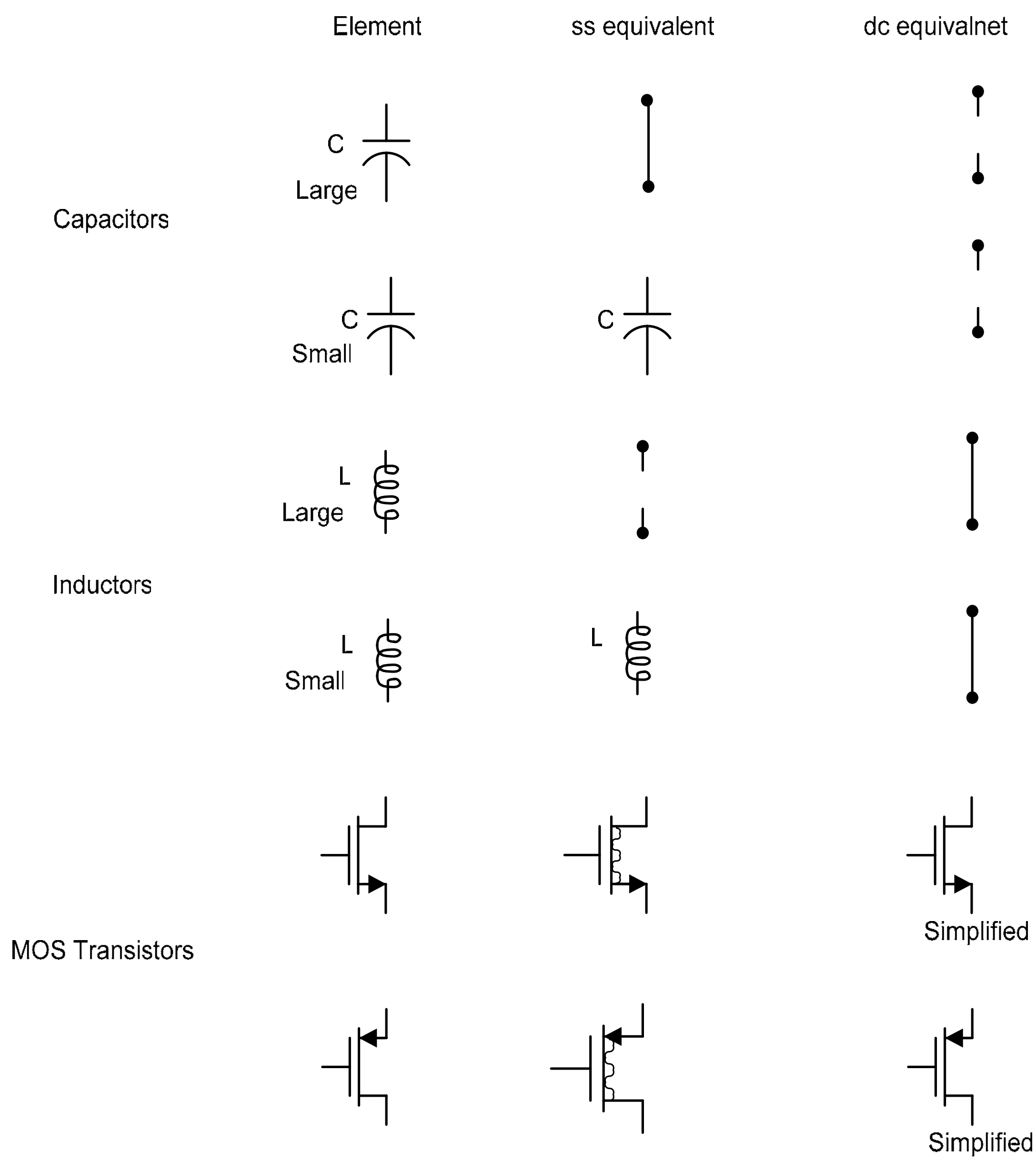
Review from Last Time:

Dc and small-signal equivalent elements



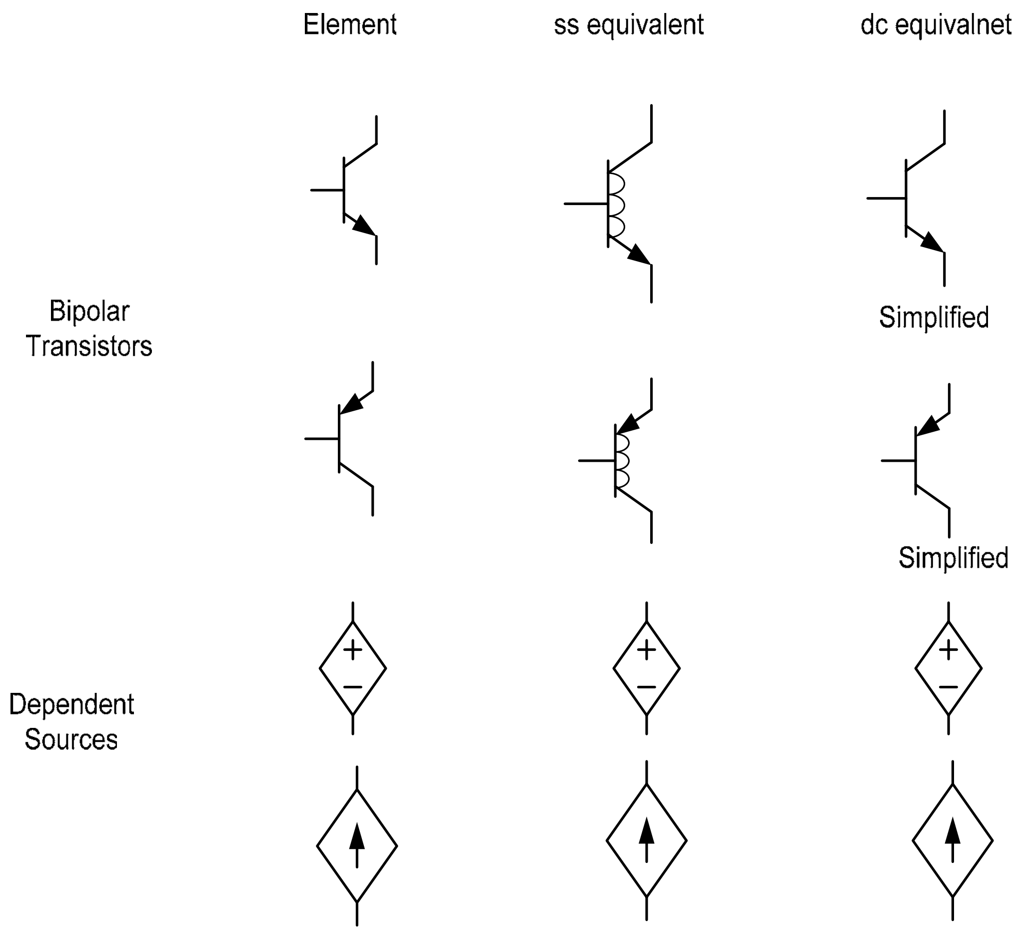
Review from Last Time:

Dc and small-signal equivalent elements



Review from Last Time:

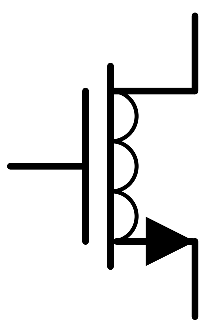
Dc and small-signal equivalent elements

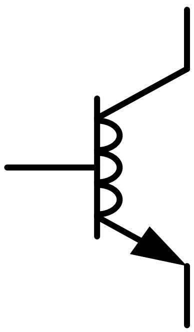


Review from Last Time:

How is the small-signal equivalent circuit obtained from the nonlinear circuit?

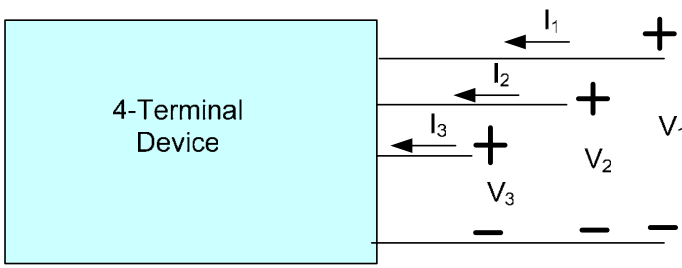
What is the small-signal equivalent of the MOSFET and BJT ?





Review from Last Time:

4-terminal small-signal network summary

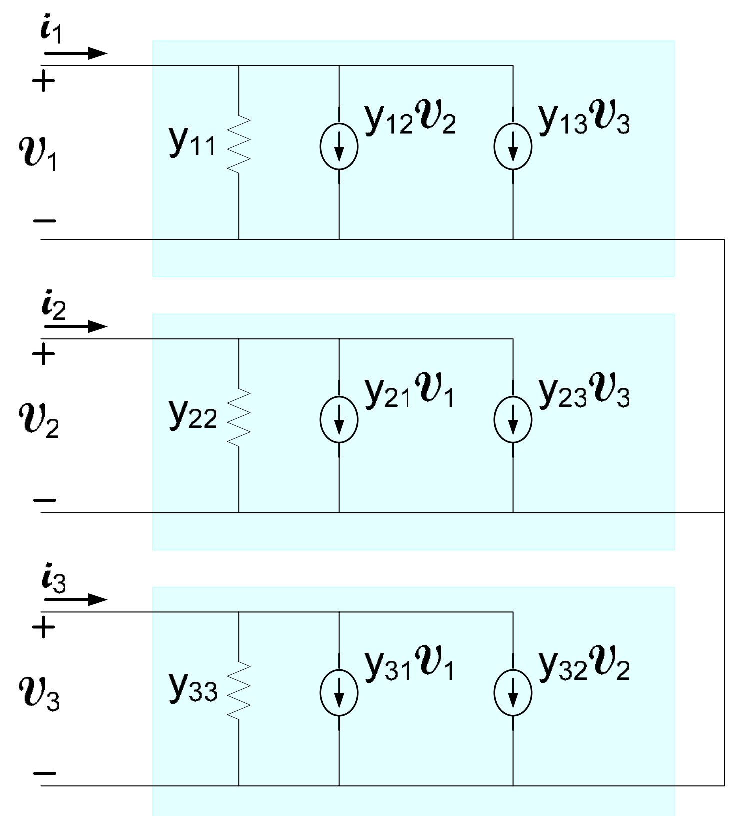


$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Small signal model:

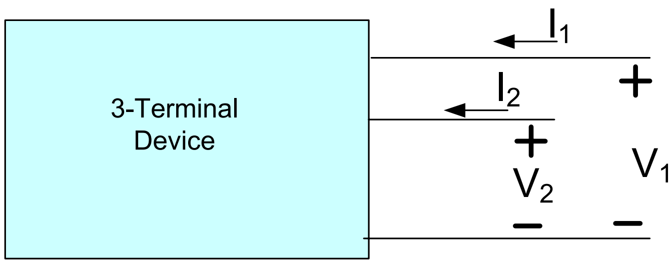
$$\boxed{\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \\ i_2 &= y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \\ i_3 &= y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \end{aligned}}$$

$$y_{ij} = \left.\frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j}\right|_{\vec{V}=\vec{V}_Q}$$



Review from Last Time:

3-terminal small-signal network summary

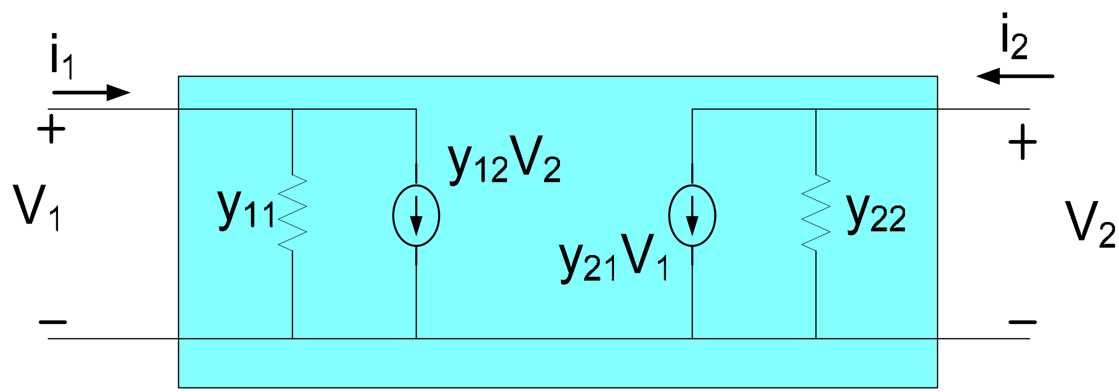


$$\left.\begin{aligned} \mathbf{I_1} &= \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2}) \\ \mathbf{I_2} &= \mathbf{f_2}(\mathbf{V_1},\mathbf{V_2}) \end{aligned}\right\}$$

Small signal model:

$$\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 \\ i_2 &= y_{21} v_1 + y_{22} v_2 \end{aligned}$$

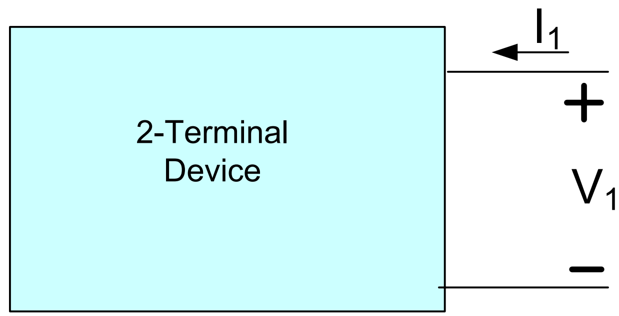
$$\mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}}$$



Review from Last Time:

2-terminal network summary

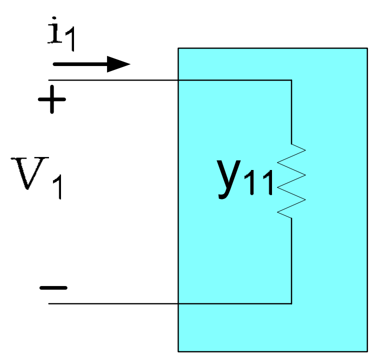
Small-Signal Model



$$\boxed{i_1 = y_{11} v_1}$$

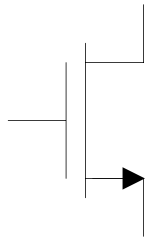
$$y_{11} = \left.\frac{\partial f_1(V_1)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q} \qquad \vec{V} = V_{1Q}$$

A Small Signal Equivalent Circuit



Review from Last Time:

Small Signal Model of MOSFET

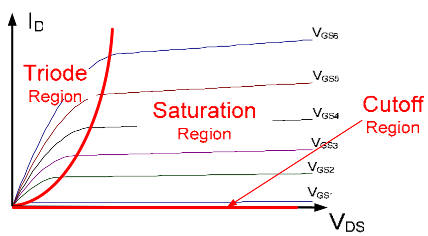


Large Signal Model

$$I_G = 0$$

3-terminal device

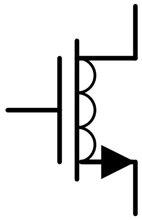
$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX}\dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX}\dfrac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$



MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region

Review from Last Time:

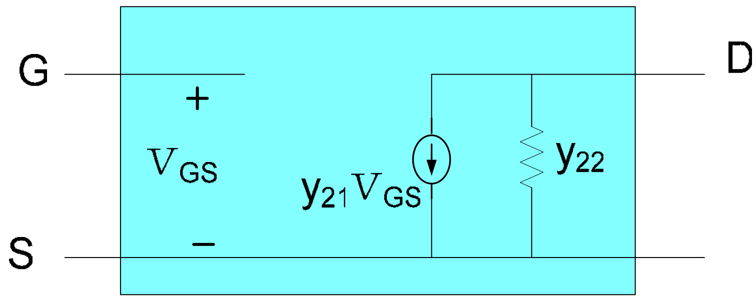
Small Signal Model of MOSFET

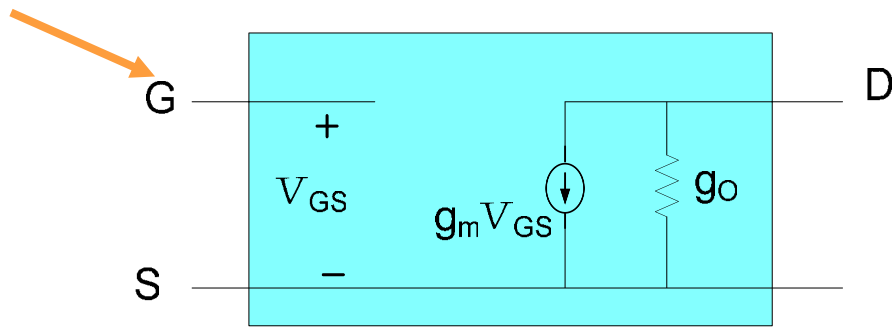


by convention, $y_{21}=g_m$, $y_{22}=g_0$

$$y_{21} \cong g_m = \mu C_{OX}\frac{W}{L}\left(V_{GSQ}-V_T\right)$$

$$y_{22} = g_o \cong \lambda I_{DQ}$$

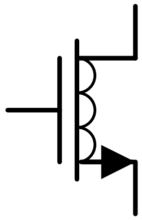




$$i_G = 0$$
$$i_D = g_m v_{GS} + g_o v_{DS}$$

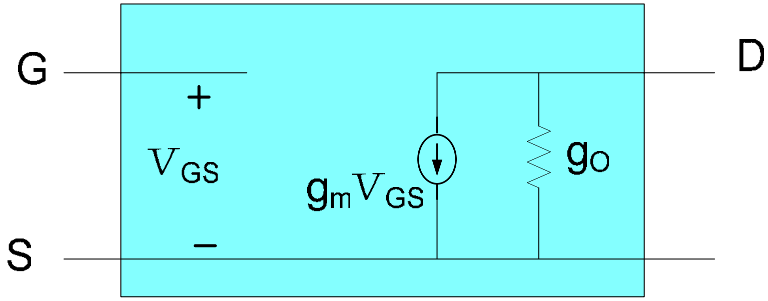
Review from Last Time:

Small Signal Model of MOSFET



$$g_m = \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

$$g_o \cong \lambda I_{DQ}$$



Alternate equivalent expressions:

$$I_{DQ} = \mu C_{OX} \frac{W}{2L}(V_{GSQ} - V_T)^2 (1 + \lambda V_{DSQ}) \cong \mu C_{OX} \frac{W}{2L}(V_{GSQ} - V_T)^2$$

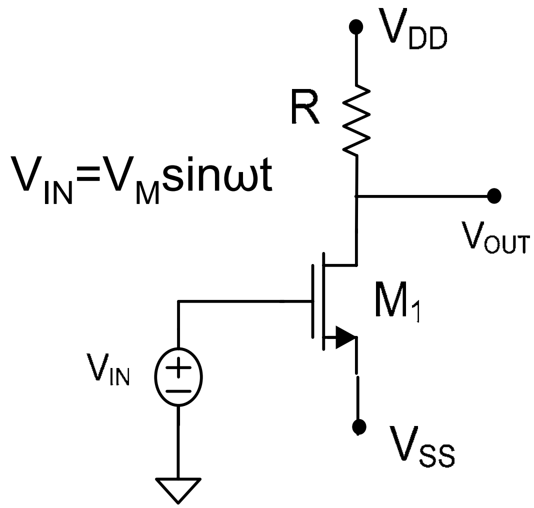
$$g_m = \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

$$g_m = \sqrt{2\mu C_{OX} \frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

Review from Last Time:

Small signal analysis example



$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

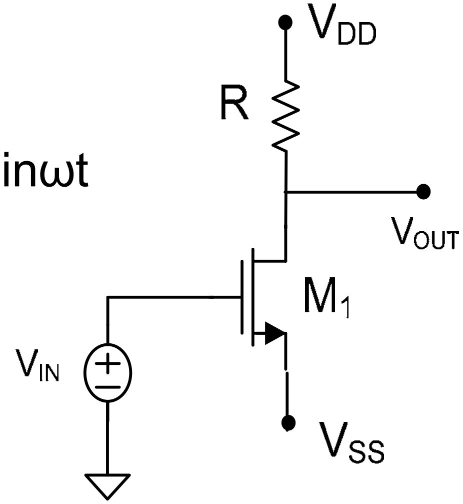
Observe the small signal voltage gain is twice the Quiescent voltage across R divided by $V_{SS}+V_T$

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

Consider again:

Small signal analysis example

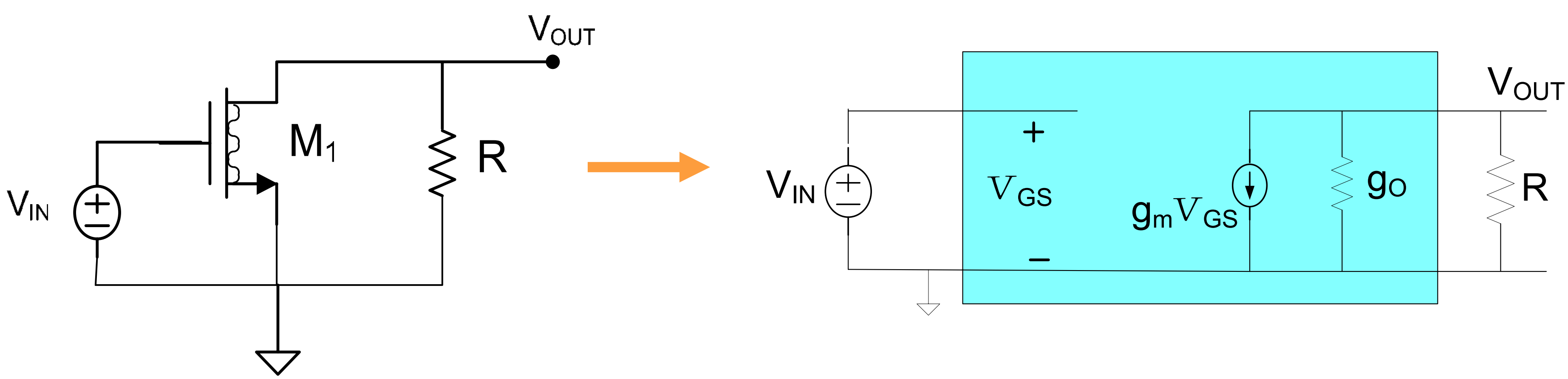
$V_{IN}=V_M \sin\omega t$



$$A_V = \frac{2 I_{DQ} R}{\left[V_{SS} + V_T\right]}$$

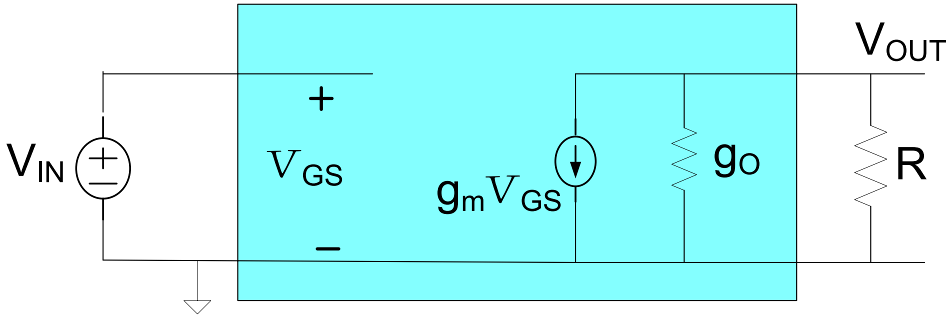
Derived for λ=0

$$I_{DQ} = \mu C_{OX} \frac{W}{2L} \left(V_{GSQ} - V_T\right)^2$$



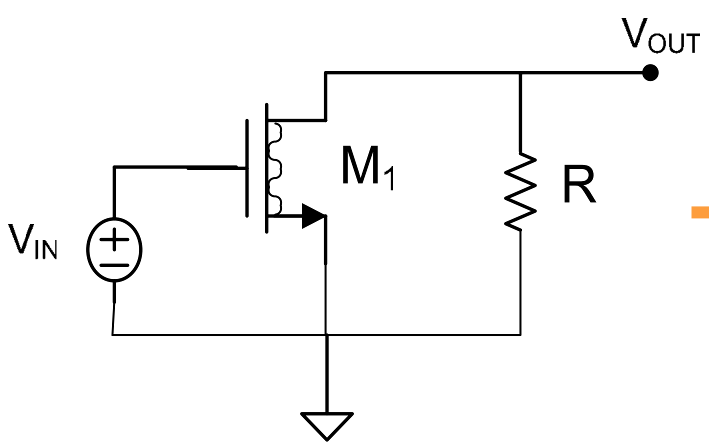
Consider again:

Small signal analysis example



$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_O + 1/R}$$

For $\lambda=0$, $g_O = \lambda I_{DQ} = 0$



$$A_V = \frac{V_{OUT}}{V_{IN}} = -g_m R$$

but

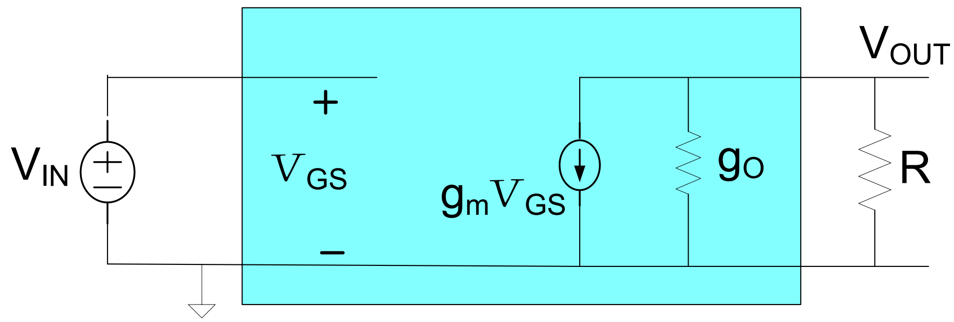
$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} \qquad V_{GSQ} = -V_{SS}$$

thus

$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

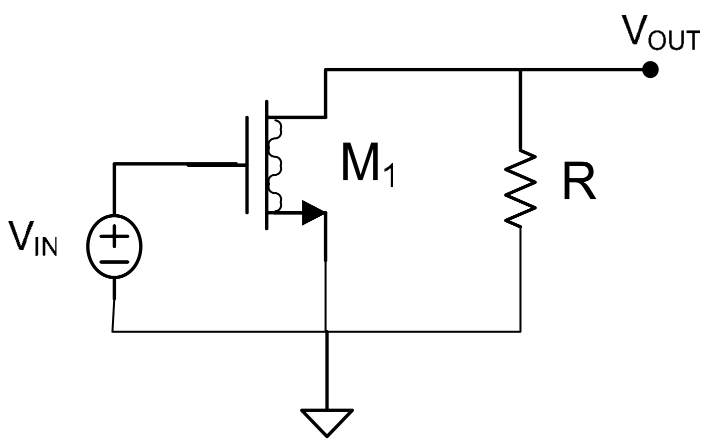
Consider again:

Small signal analysis example



$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

For λ=0, g_O= λ_{IDQ} = 0



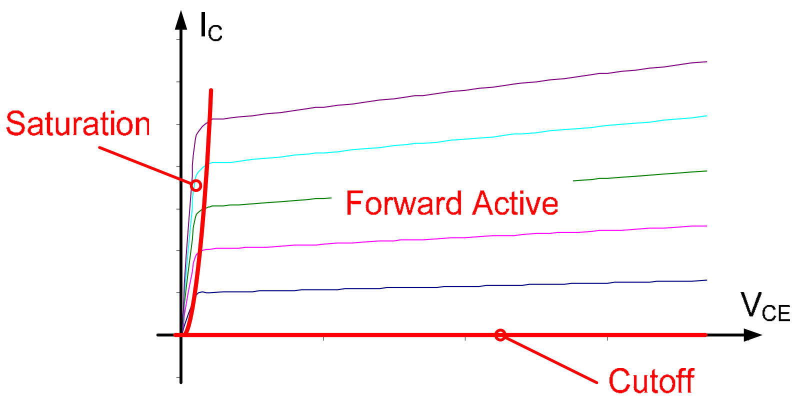
$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

Same expression as derived before

More accurate gain can be obtained if
λ effects are included and does not significantly
increase complexity of small signal analysis

Small Signal Model of BJT

3-terminal device



Forward Active Model:

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$\mathbf{I_B} = \frac{\mathbf{J_S A_E}}{\boldsymbol{\beta}} \mathbf{e}^{\frac{\mathbf{V_{BE}}}{\mathbf{V_t}}}$$

Usually operated in Forward Active Region when small-signal model is needed

Small Signal Model of BJT

$$I_1 = f_1\left(V_1, V_2\right) \quad \Longleftrightarrow \quad I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_2 = f_2\left(V_1, V_2\right) \quad \Longleftrightarrow \quad I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:

$$y_{ij} = \left.\frac{\partial f_i\left(V_1, V_2\right)}{\partial V_j}\right|_{\vec{V}=\vec{V}_Q}$$

$$y_{11} = g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} \qquad y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q}$$

$$y_{21} = g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} \qquad y_{22} = g_o = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q}$$

Small Signal Model of BJT

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \qquad I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:

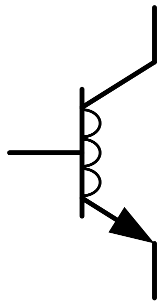
$$g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \frac{1}{V_t} \frac{J_S A_E}{\beta} \left. e^{\frac{V_{BE}}{V_t}} \right|_{\vec{V}=\vec{V}_Q} = \frac{I_{BQ}}{V_t} \cong \frac{I_{CQ}}{\beta V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{21} = g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \frac{1}{V_t} J_S A_E e^{\frac{V_{BE}}{V_t}} \left.\left(1 + \frac{V_{CE}}{V_{AF}}\right)\right|_{\vec{V}=\vec{V}_Q} = \frac{I_{CQ}}{V_t}$$

$$y_{22} = g_o = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{J_S A_E e^{\frac{V_{BE}}{V_t}}}{V_{AF}}\right|_{\vec{V}=\vec{V}_Q} \cong \frac{I_{CQ}}{V_{AF}}$$

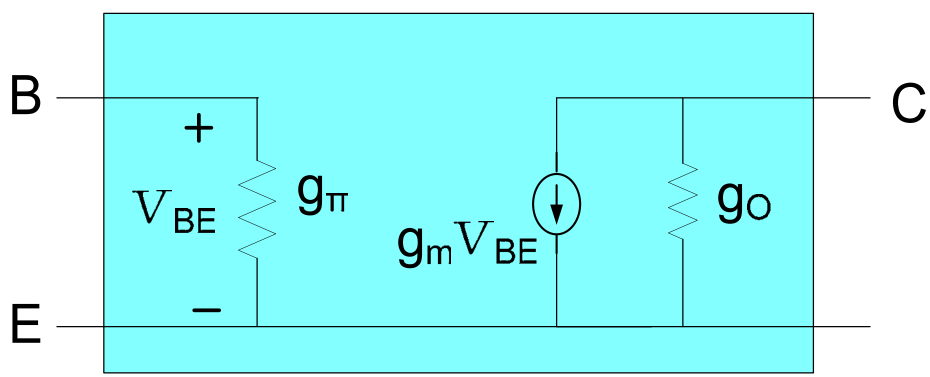
Small Signal Model of BJT



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$
$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$i_B = g_\pi v_{BE}$$
$$i_C = g_m v_{BE} + g_o v_{CE}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_m = \frac{I_{CQ}}{V_t} \qquad g_o = \frac{I_{CQ}}{V_{AF}}$$



Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Recall:

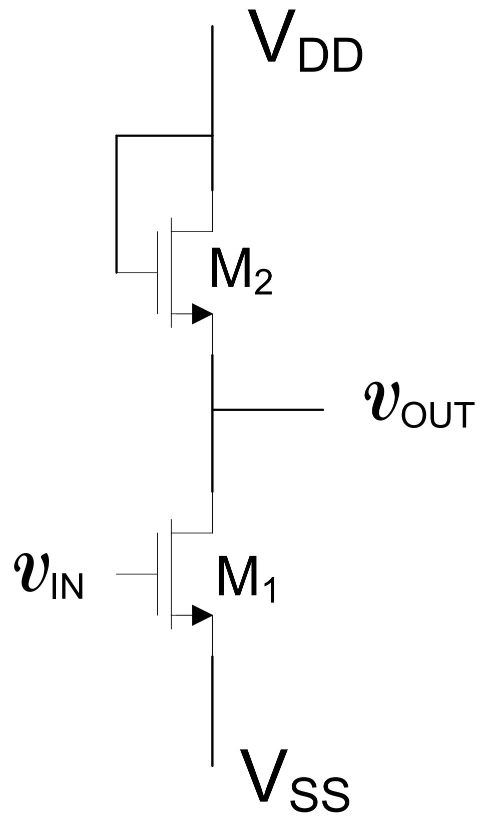
Alternative Approach to small-signal analysis of nonlinear networks

1. **Linearize nonlinear devices**
   (have small-signal model for key devices!)

2. **Replace all devices with small-signal equivalent**

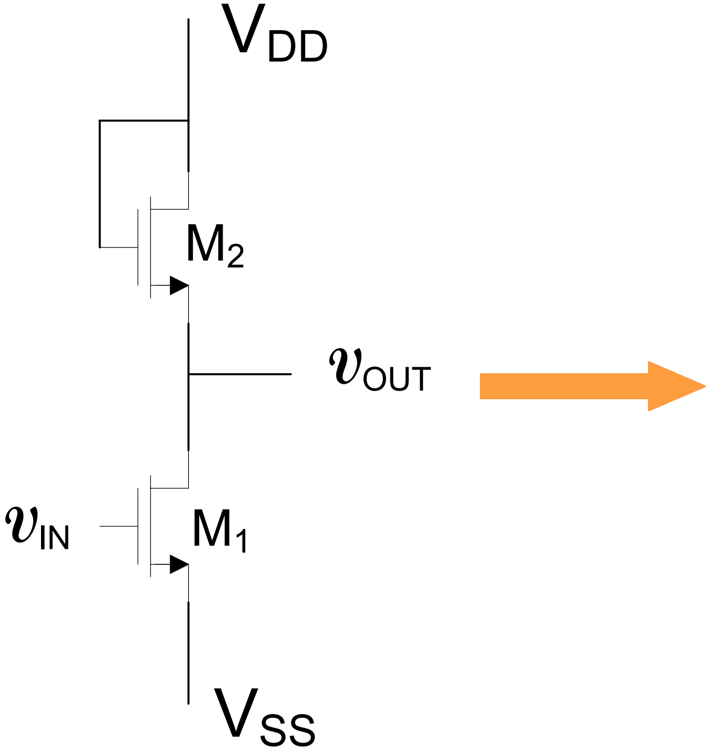
***3.* Solve linear small-signal network**

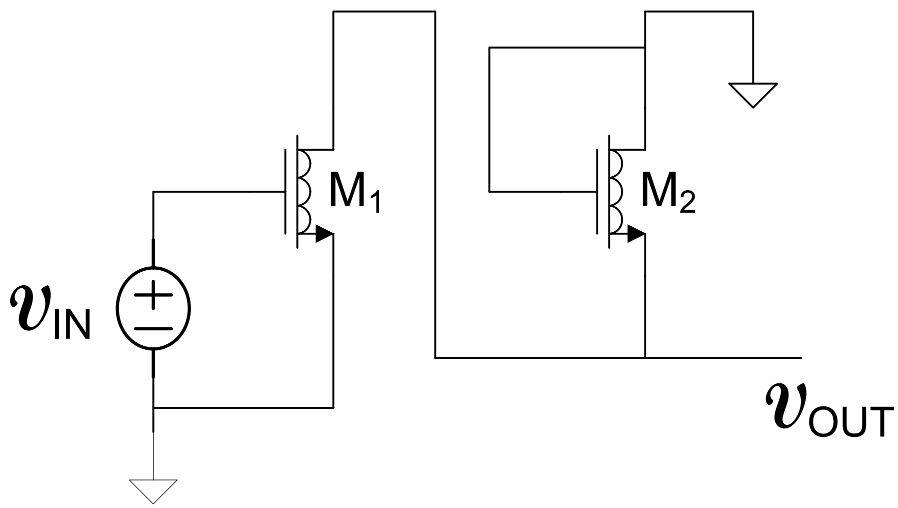
Example:



Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. *Assume* M_1 *and* M_2 *are operating in the saturation region and that* $\lambda = 0$

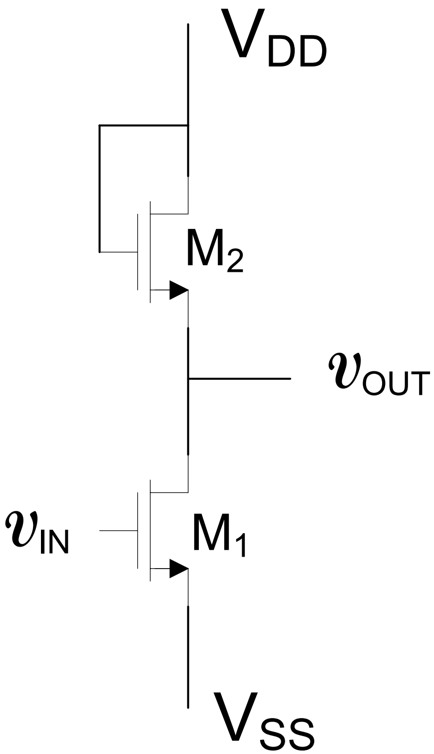
Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$*

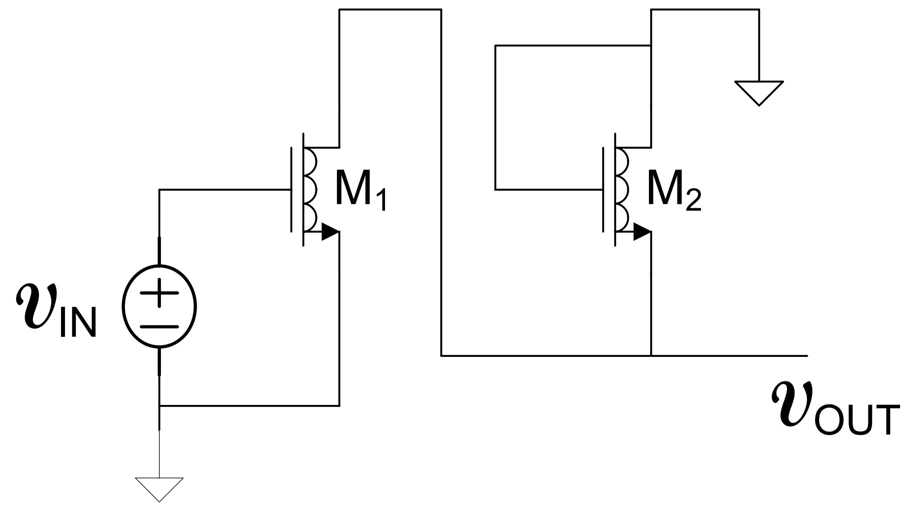




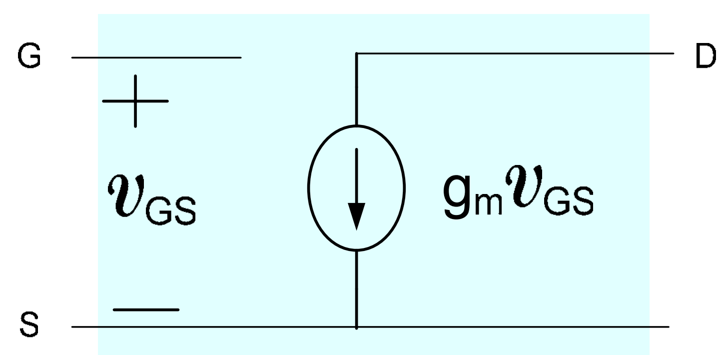
Small-signal circuit

Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that λ=0*



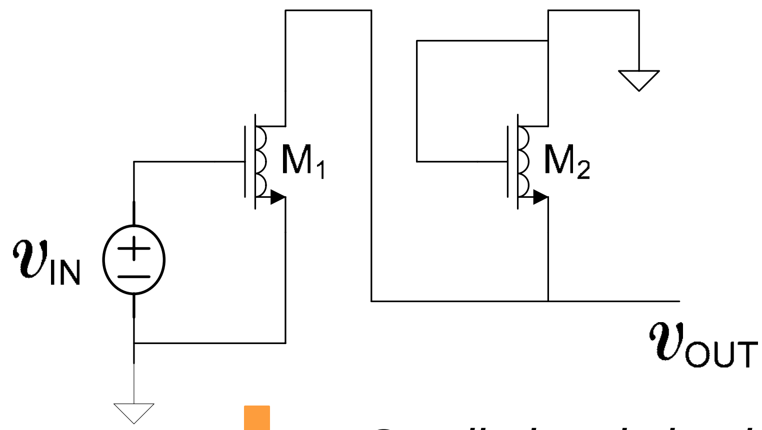


Small-signal circuit

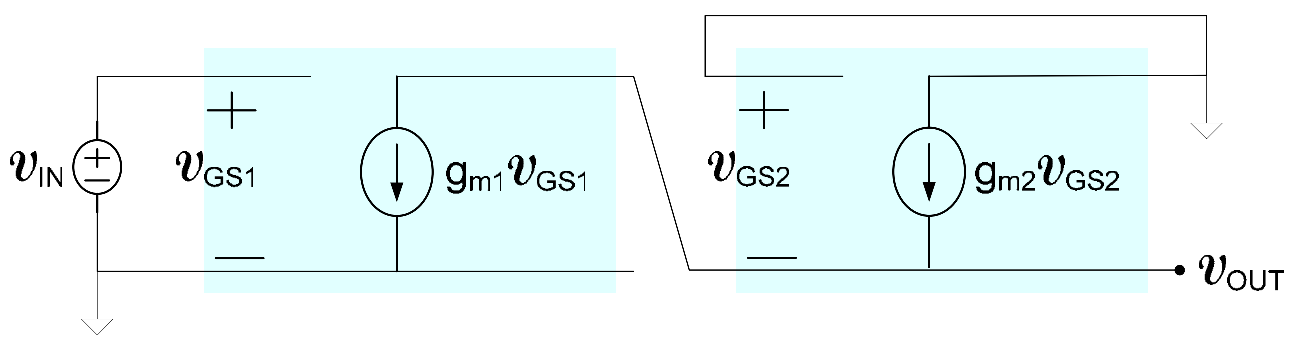


Small-signal MOSFET model for λ=0

Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$*

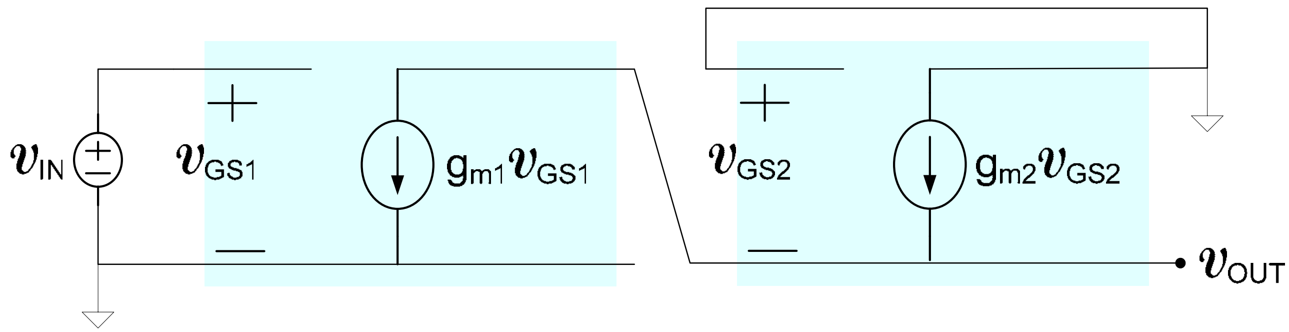


Small-signal circuit



Small-signal circuit

Example:



Small-signal circuit

Analysis:

By KCL

$$g_{m1}v_{GS1} = g_{m2}v_{GS2}$$

but

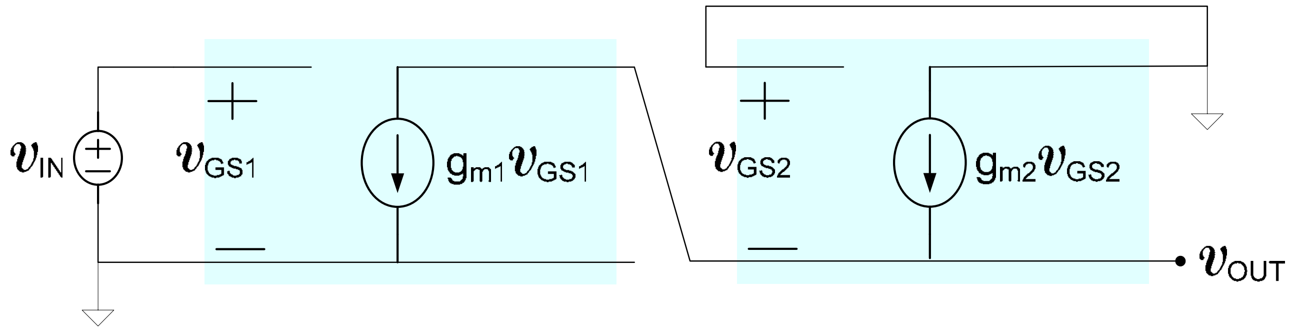
$$v_{GS1} = v_{IN}$$

$$-v_{GS2} = v_{OUT}$$

thus:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}}$$

Example:



Small-signal circuit

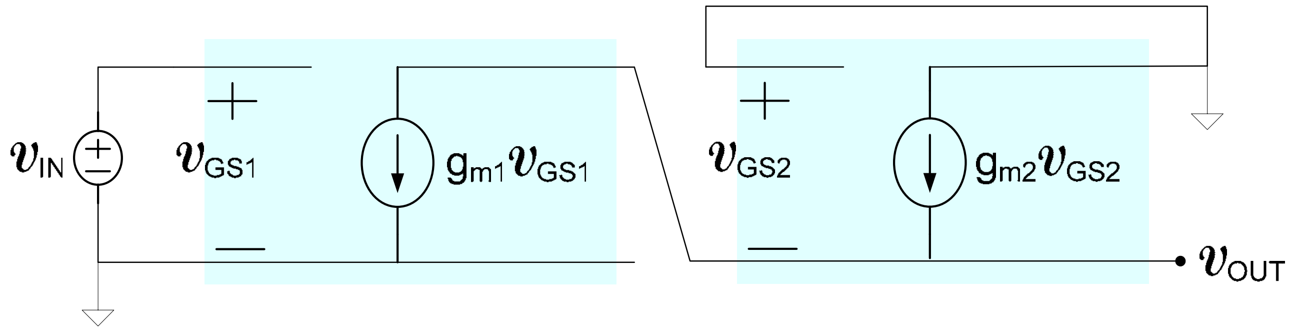
Analysis:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}}$$

Recall: $$g_m = -\sqrt{2I_D\mu C_{OX}}\sqrt{\frac{W_1}{L_1}}$$

$$A_V = -\frac{\sqrt{2I_D\mu C_{OX}\frac{W_1}{L_1}}}{\sqrt{2I_D\mu C_{OX}\frac{W_2}{L_2}}} = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}}$$

Example:



Small-signal circuit

Analysis:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}}$$

Recall:

$$A_V = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}}$$

If $L_1 = L_2$, obtain

$$A_V = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}} = -\sqrt{\frac{W_1}{W_2}}$$

The width and length ratios can be accurately set when designed in a standard CMOS process